

Generalized Holonomy of Supergravities with 8 Real Supercharges¹

A. Batrachenko² and W. Y. Wen³

Michigan Center for Theoretical Physics

*Randall Laboratory, Department of Physics, University of Michigan
Ann Arbor, MI 48109-1120, USA*

Abstract

We show that the generalized holonomy groups of ungauged supergravity theories with 8 real supercharges must be contained in $SL(2-\nu, \mathbb{H}) \ltimes \nu \mathbb{H}^{2-\nu} \subseteq SL(2, \mathbb{H})$, where $SL(2, \mathbb{H})$ is the generalized structure group. Here $n = 4\nu$ is the number of preserved supersymmetries, so the allowed values are limited to $n = 0, 4, 8$. In particular, solutions of ungauged supergravities in four, five and six dimensions are examined and found to explicitly follow this pattern. We also argue that the G -structure has to be a subgroup of this generalized holonomy group, which may provide a possible classification for supergravity vacua with respect to the number of supercharges.

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²abat@umich.edu

³wenw@umich.edu

1 Introduction

For a general solution of supergravity theory in dimension D , the number of supersymmetries preserved by this background depends on the number of covariantly constant spinors,

$$\mathcal{D}_M \varepsilon = 0, \quad (1)$$

Generically, \mathcal{D}_M includes the covariant derivative D_M with Levi-Civita connection ω and contribution from flux F . For the simpler vacua where flux vanishes, one obtains the integrability condition,

$$[D_M, D_N] \varepsilon = \frac{1}{4} R_{MN}{}^{AB} \Gamma_{AB} \varepsilon = 0 \quad (2)$$

where $R_{MN}{}^{AB}$ is Riemann tensor and Γ_{AB} can be viewed as generators of structure group $\mathcal{G} = SO(1, D-1)$. The holonomy group \mathcal{H} associated with ω is a subgroup of \mathcal{G} . The number of preserved supersymmetries is the number of singlets appearing in the decomposition of the spinor representation of $SO(1, D-1)$ under \mathcal{H} . The holonomy groups in Lorentzian signature has been studied [1] and in Euclidean signature they have been completely classified [2].

For generic backgrounds, however, one has to consider the contribution from flux; hence the classification above is no longer valid. A statement that an enlarged structure group $\tilde{\mathcal{G}}$ and a generalized holonomy group $\tilde{\mathcal{H}}$ exist to play similar roles as in the pure (pseudo) Riemannian background has been verified for M-theory vacua in eleven dimensions [3, 4, 5, 6, 7, 8] and type IIB theory in ten dimensions [9]. It would be interesting to see if this approach could be applied to supergravity theories in lower dimensions and if the holonomy groups of theories in different dimensions are related. In this paper, we argue that a single generalized structure group, with corresponding generalized holonomy subgroups is responsible for classification of supergravity vacua with 8 real supercharges. In particular, all solutions of ungauged supergravity in four, five, and six dimensions are investigated to support our proposition. While the G -structure is useful to construct solutions, the generalized holonomy approach may provide a better classification of supersymmetry vacua with regard to preserved supercharges, at least for the vacua examined in this paper.

This paper is organized as follows. In section 2, we show that the generalized structure group for supergravities with 8 supercharges in $D = 4, 5, 6$ is $SL(2, \mathbb{H})$. Then we argue that the generalized holonomy is at least contained in $SL(1, \mathbb{H}) \ltimes \mathbb{H}$ for all 1/2-BPS solutions. In section 3, as an example, we examine the generalized holonomy of minimal five-dimensional

supergravity solutions of both timelike and null cases. Finally, investigation of four- and six-dimensional supergravity vacua provides more evidence and this discussion is carried out in section 4.

2 Generalized holonomy of supergravity solutions with 8 real supercharges

Here we are interested in supergravity theories with 8 real supercharges, specifically, $\mathcal{N} = (1, 0)$ in $D = 6$, $\mathcal{N} = 2$ in $D = 5$, $\mathcal{N} = 2$ in $D = 4$ dimensions. They are closely related in the sense that the last two theories can be obtained from the first one through Kaluza-Klein reduction with consistent truncation [10, 11]. The (pseudo) Riemannian structure group is generated by gamma matrices with two indices, denoted as $\Gamma^{(2)}$, as shown in (2), which is $SO(1, D-1)$ for each $D = 4, 5, 6$. In the more general case of non-zero flux F , one considers the generalized structure group using the Killing spinor equation with flux F turned on, which can be written schematically as

$$\mathcal{D}_M \varepsilon = [D_M + (\Gamma^{(p+1)} F_{(p)})_M + (\Gamma^{(p-1)} F_{(p)})_M] \varepsilon, \quad (3)$$

where $F_{(p)}$ is a two-form in $D = 4, 5$ and a three-form in $D = 6$. Hence, in (3) we find the combinations $\Gamma^{(1)}, \Gamma^{(2)}, \Gamma^{(3)}$ for $D = 4, 5$ and $\Gamma^{(2)}, \Gamma^{(4)}$ for $D = 6$.

One can compute the independent gamma matrix combinations for each specific dimension. For $D = 4$, choosing the chirality projector $\Gamma^5 \varepsilon = \varepsilon$, the integrability condition gives one more generator $\Gamma^{(4)}$, and as a result 15 independent generators in total. For $D = 5$, the fact $\Gamma^{(2)}$ is dual to $\Gamma^{(3)}$ gives us exactly 15 generators. As to $D = 6$ case, by choosing the chirality projector $\Gamma^7 \varepsilon = \varepsilon$, one only obtains 15 relevant generators for the $\mathcal{N} = (1, 0)$ theory. The 15 in each of the three cases forms a real Clifford group isomorphic to $SL(2, \mathbb{H})$. Hence this is the generalized structure group with regard to 8 supercharges, at least for the three theories just mentioned.

To find the generalized holonomy group for vacua preserving a subset $0 \leq n \leq 8$ of supersymmetries, one considers the subgroup of $SL(2, \mathbb{H})$ that stabilizes a quaternion-valued spinor of the form $(s^1 \quad s^2)$, where each s^i has four real components. Consider, for example,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} s^1 \\ s^2 \end{pmatrix}, \quad (4)$$

Spinor	Maximal supersymmetry	Maximal generalized holonomy
Real	32	$SL(32 - n, \mathbb{R}) \ltimes n\mathbb{R}^{32-n}$
Complex	8	$GL(\frac{8-n}{2}, \mathbb{C}) \ltimes \frac{n}{2}\mathbb{C}^{\frac{8-n}{2}}$
Quaternion	8	$SL(\frac{8-n}{4}, \mathbb{H}) \ltimes \frac{n}{4}\mathbb{H}^{\frac{8-n}{4}}$

Table 1: Generalized holonomies of supergravities with 8 and 32 real supercharges. Here n is the number of preserved supersymmetries.

where $A, B, C, D \in \mathbb{H}$. We see that for arbitrary A, B, C, D there is no preserved spinor ($n = 0$). On the other hand, the trivial identity ($A = D = 1, B = C = 0$) will preserve both s^1 and s^2 ($n = 8$). The only nontrivial choice is $A = 1, C = 0$ and $D \in SL(1, \mathbb{H})$, which preserves half the supercharges ($n = 4$), say s^1 in this choice. This demonstrates that the number of preserved supersymmetries is restricted to the values $n = 0, 4, 8$, corresponding to solutions with no supersymmetry, half and maximal supersymmetry, respectively. This observation agrees with [12] in $D = 5$ ungauged supergravity case. In conclusion, for a solution to preserve exactly n supersymmetries the generalized holonomy \mathcal{H} has to satisfy the condition $SL(\frac{4-n}{4}, \mathbb{H}) \ltimes \frac{n+4}{4}\mathbb{H}^{\frac{4-n}{4}} \subset \mathcal{H} \subseteq SL(\frac{8-n}{4}, \mathbb{H}) \ltimes \frac{n}{4}\mathbb{H}^{\frac{8-n}{4}}$.

To complete our discussion here, we note that the $SL(2, \mathbb{H})$ classification only holds for the ungauged supergravities. For gauged supergravities, a gauge term such as igA_μ , with coupling g , has to be included in the generalized covariant derivative (3). This modifies the generalized structure group to the complex Clifford group $GL(4, \mathbb{C})$ with 32 generators. However, as we have seen above, it is restricted to the real Clifford group $SL(2, \mathbb{H})$ in the ungauged theory.

Table 1 summarizes our findings together with another fundamental result for the generalized holonomy of M-theory [6, 7]. The first category in table 1 is useful for discussion of supergravity theories with 32 real supercharges or fewer [8] where the spinors take values in \mathbb{R} . The second and third categories are useful for discussion of supergravity with 8 real supercharges or less where the spinors may take values in either \mathbb{C} or \mathbb{H} . To be specific, we may classify the generalized holonomy of $D = 5$ gauged supergravity solutions according to the second category. We will return to this problem later in section 4.

In the following section, as an example, we will explicitly compute the generalized holonomy group for $D = 5$ minimal supergravity to support the classification scheme of table 1.

3 Generalized holonomy of $D = 5$ minimal supergravity

3.1 $D = 5$ minimal supergravity

All supersymmetric solutions of minimal supergravity in five dimensions have been classified using the idea of G -structure [12]. Here we follow their convention and give a brief review as follows. The bosonic action for minimal supergravity in five dimensions is

$$S = \frac{1}{4\pi G} \int -\frac{1}{4} R \star 1 - \frac{1}{2} F \wedge \star F - \frac{2}{3\sqrt{3}} F \wedge F \wedge A, \quad (5)$$

If a solution to the equation of motion derived from the above action is supersymmetric, it admits a covariantly constant spinor satisfying

$$\mathcal{D}_M \varepsilon^a = [D_M + \frac{1}{4\sqrt{3}} (\Gamma_M^{NP} - 4\delta_M^N \Gamma^P) F_{NP}] \varepsilon^a = 0, \quad (6)$$

where ε^a is a symplectic Majorana Killing spinor with 8 real components. Out of this Killing spinor, one can construct a real scalar f , a real 1-form V and three complex 2-forms Φ as

$$\begin{aligned} f \epsilon^{ab} &= \bar{\varepsilon}^a \varepsilon^b, \\ V_M \epsilon^{ab} &= \bar{\varepsilon}^a \Gamma_M \varepsilon^b, \\ \Phi_{MN}^{ab} &= \bar{\varepsilon}^a \Gamma_{MN} \varepsilon^b, \end{aligned} \quad (7)$$

There are also algebraic and differential constraints for these objects to satisfy, which we will not detail here. It turns out that V_M is a Killing form (vector) satisfying

$$V_M V^M = f^2, \quad (8)$$

and the solutions can be classified according to whether V_M is timelike ($f \neq 0$) or null ($f = 0$). In the following, we examine both cases.

3.2 The timelike solution

By choosing the timelike killing vector $V = \partial/\partial t$, the metric, in general, can be written locally as [12]

$$ds^2 = f^2 (dt^2 + \omega)^2 - f^{-1} h_{ij} dx^i dx^j, \quad (9)$$

where h_{ij} , $i, j = 1..4$ is the metric of a hyper-Kähler base manifold \mathcal{B} together with a globally defined f and locally defined 1-form connection ω on \mathcal{B} . The two form $d\omega$ can then be split

into self-dual and anti-self-dual parts with respect to h_{ij} :

$$fd\omega = G^+ + G^- \quad (10)$$

It is convenient to introduce a local (flat) frame such that

$$\begin{aligned} e^0 &= f(dt + \omega), \\ e^i e^j &= f^{-1} h_{ij} dx^i dx^j \end{aligned} \quad (11)$$

Then the two-form flux can be written as

$$F = \frac{\sqrt{3}}{2} de^0 - \frac{1}{\sqrt{3}} G^+. \quad (12)$$

The Bianchi condition and equation of motion give the constraints:

$$\begin{aligned} dG^+ &= 0, \\ \Delta f^{-1} &= \frac{2}{9} G^+{}_{ij} G^{+ij}, \end{aligned} \quad (13)$$

where Δ is the Laplacian with respect to \mathcal{B} .

Using the metric given by (9) and the constraints above, we can obtain the Killing spinor equations:

$$\mathcal{D}_t \varepsilon = [\partial_t + f^{1/2} \partial_i f \gamma^i P_- - \frac{1}{6} f^2 G^+ \cdot \gamma P_-] \varepsilon, \quad (14)$$

$$\begin{aligned} \mathcal{D}_i \varepsilon &= [\partial_i - \frac{1}{2} f^{-1} \partial_i f + \omega_i (f^{1/2} \partial_j f \gamma^j - \frac{1}{6} f^2 G^+ \cdot \gamma) P_- \\ &\quad - \frac{1}{2} f^{-1} \partial_j f (\gamma_i{}^j - 2\delta_i^j) P_- - \frac{1}{3} f^{1/2} (G^+ + 3G^-)_{ij} \gamma^j P_-] \varepsilon, \end{aligned} \quad (15)$$

where the gamma matrices are in the local frame and obey

$$\begin{aligned} \{\gamma_A, \gamma_B\} &= 2\eta_{AB}, \\ \gamma_{ABCDE} &= \epsilon_{ABCDE}. \end{aligned} \quad (16)$$

Here we choose $\eta_{00} = 1$, $\eta_{ij} = -\delta_{ij}$, and $\epsilon_{01234} = 1$. $P_{\pm} = \frac{1}{2}(1 \pm \gamma^0)$ is the $\frac{1}{2}$ -BPS projection for the timelike background. To obtain the generalized holonomy, we examine the commutator of covariant derivatives. Defining

$$\mathcal{M}_{MN} \varepsilon = [\mathcal{D}_M, \mathcal{D}_N] \varepsilon, \quad (17)$$

we find

$$\begin{aligned}\mathcal{M}_{it\varepsilon} &= \mathcal{A}_{ij}\gamma^j P_- + \mathcal{B}_{ijk}\gamma^{jk} P_- \varepsilon, \\ \mathcal{M}_{ij\varepsilon} &= \mathcal{C}_{ijm}\gamma^m P_- + \mathcal{D}_{ijmn}\gamma^{mn} P_- \varepsilon,\end{aligned}\tag{18}$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ only depend on functions f and G . For example, \mathcal{A} and \mathcal{B} are given by

$$\begin{aligned}\mathcal{A} &= f^{1/2}\partial_i\partial_j f - \frac{1}{2}f^{-1/2}\delta_{ij}(\partial f)^2 + \frac{2}{9}f^{5/2}(G^+ + 3G^-)_{ik}G^+_{~j}{}^k \\ \mathcal{B} &= -\frac{1}{6}f^2\partial_i G^+_{~jk} - \frac{1}{3}[f\partial_{(i}fG^+_{~j)k} - \delta_{ij}\partial_l fG^+_{~lk}].\end{aligned}\tag{19}$$

Since the details are not important for finding the generalized holonomy, we do not provide the more complicated expressions for \mathcal{C} and \mathcal{D} .

We see that the only combination of gamma matrices showing up in \mathcal{M}_{MN} are given by $\gamma^i P_-$ and $\gamma^{ij} P_-$. Defining two sets of generators

$$\begin{aligned}T^{ij} &= -\frac{i}{2}P_- \gamma^{ij} P_-, \\ K^i &= P_+ \gamma^i P_-, \end{aligned}\tag{20}$$

and observing that $\frac{1}{2}\epsilon_{ijkl}\gamma^{kl} = -\gamma_{ij}\gamma^0$, we find that the only independent generators for T^{ij} are T^{12} , T^{23} and T^{31} . They generate the $SU(2)_-$ algebra, where the $-$ refers to the sign of the P_- projection. The other generators K^i obviously commute among themselves due to the projection identity $P_+ P_- = 0$. If we choose T^{31} as the Cartan generator for $SU(2)$, we may see that $\{K^1, K^3\}$ and $\{K^2, K^4\}$ form two doublets. Therefore the generalized holonomy for the timelike solutions is

$$\mathcal{H}_{timelike} = SU(2)_- \ltimes 2\mathbb{R}^2.\tag{21}$$

3.3 The null solution

When the Killing vector V defined in (7) is null it is possible to choose coordinates (u, v, y_i) , $i=1,2,3$, such that V is tangent to geodesics in the surface of constant u with affine parameter v , i.e. $V = \frac{\partial}{\partial v}$. In this set of coordinates the most general metric obeying the algebraic identities that relate the components of the forms (7) is [12]

$$ds^2 = H^{-1}(u, \mathbf{x})(\mathcal{F}(u, \mathbf{x})du^2 + 2dudv) - H^2(u, \mathbf{x})(d\mathbf{x} + \mathbf{a}(u, \mathbf{x})du)^2.\tag{22}$$

The field strength is given by

$$F_{ui} = -\frac{H^{-2}}{4\sqrt{3}}\epsilon_{ijk}\partial_j(H^3a_k), \quad (23)$$

$$F_{ij} = -\frac{\sqrt{3}}{4}\epsilon_{ijk}\partial_k H. \quad (24)$$

The generalized covariant derivative in this background takes the form

$$\mathcal{D}_v\varepsilon = \left(\partial_v + \frac{1}{2}H^{-2}\partial_a H\gamma^{a+}P_-\right)\varepsilon, \quad (25)$$

$$\mathcal{D}_u\varepsilon = \left[(\partial_u - a^i\partial_i) - \frac{1}{4}\partial_i(\mathcal{F}H^{-1})\gamma^{i+}P_- + \frac{1}{3}\epsilon_{ijk}\partial_j a_k\gamma^i P_-\right]\varepsilon, \quad (26)$$

$$\begin{aligned} \mathcal{D}_i\varepsilon = & \left[\partial_i - H^{-1}\epsilon_{ijk}\partial_j H\gamma^k P_- - \frac{1}{6}H^2\epsilon_{ijk}\partial_j a_k\gamma^+ P_- \right. \\ & \left. + \left(\frac{1}{3}H^2\partial_j a_i + \frac{1}{6}H^2\partial_i a_j - \frac{1}{2}\delta_{ij}H(\partial_u H - a_k\partial_k H)\right)\gamma^{j+}P_-\right]\varepsilon. \end{aligned} \quad (27)$$

Here the $\frac{1}{2}$ -BPS projectors are defined as $P_- = \frac{1}{2}\gamma^-\gamma^+$, $P_+ = \frac{1}{2}\gamma^+\gamma^-$. The gamma matrices are defined in (16) where $\eta_{+-} = \eta_{-+} = 1$, $\eta_{ij} = -\delta_{ij}$ and $\epsilon_{+-123} = 1$.

The action of the holonomy group on an arbitrary spinor ε is represented by the commutator of the generalized covariant derivatives as defined in (17)

$$\mathcal{M}_{vi}\varepsilon = \mathcal{A}_{ij}\gamma^{j+}\varepsilon, \quad (28)$$

$$\mathcal{M}_{vu}\varepsilon = \mathcal{B}_j\gamma^{j+}\varepsilon, \quad (29)$$

$$\mathcal{M}_{ui}\varepsilon = (\mathcal{C}_{ij}\gamma^j P_- + \mathcal{E}\gamma^+ + \mathcal{G}_{ij}\gamma^{j+})\varepsilon, \quad (30)$$

$$\mathcal{M}_{ij}\varepsilon = (\mathcal{I}_{ijk}\gamma^k P_- + \mathcal{J}\gamma^+ + \mathcal{K}_{ijk}\gamma^{k+})\varepsilon. \quad (31)$$

The expressions for quantities \mathcal{A} , \mathcal{B} , \mathcal{C} , \mathcal{E} , \mathcal{G} , \mathcal{I} , \mathcal{J} and \mathcal{K} are quite lengthy and unimportant for our problem, so we do not present them here. They involve functions $H(u, \mathbf{x})$, $\mathbf{a}(u, \mathbf{x})$ and $\mathcal{F}(u, \mathbf{x})$ and their derivatives. In particular, when the field strength vanishes, only \mathcal{B} and \mathcal{G} are nonzero and thus the holonomy group is the familiar \mathbb{R}^3 appropriate to the pseudo-Riemannian background.

In more general backgrounds the combinations of the gamma matrices involved in the commutators are $\gamma^i P_-$, γ^+ and γ^{i+} . Using this fact we define the complete set of the holonomy generators of the null solution as follows

$$T^i = -\frac{i}{2}P_-\gamma^i P_-, \quad (32)$$

$$R^i = P_+\gamma^{i+}P_-, \quad (33)$$

$$R^4 = P_+\gamma^+P_-. \quad (34)$$

The generators T^i generate an $SU(2)$ algebra because $[T^i, T^j] = i\epsilon_{ijk}T^k$. Since $(\gamma^+)^2 = 0$ the generators R^i and R^4 commute with each other forming \mathbb{R}^4 . Choosing T^3 as the Cartan generator for $SU(2)$, we find that the set of generators $\{R^i, R^4\}$ has weights $\pm\frac{1}{2}$ and thus the pairs $\{R^1, R^2\}$ and $\{R^3, R^4\}$ transform as two doublets under the action of $SU(2)$. This leads us to the conclusion that the generalized holonomy group of the null solution is

$$\mathcal{H}_{null} = SU(2) \ltimes 2\mathbb{R}^2. \quad (35)$$

4 Discussion

4.1 Relation to the G -structure

As we have seen in the previous section, the generalized holonomies of both timelike and null solutions preserving half of the supersymmetries in $D = 5$ are the same, namely $SU(2) \ltimes 2\mathbb{R}^2$. As shown in [12], the corresponding G -structures are $SU(2)$ for timelike solutions and R^3 for null ones. Both are subgroups of the generalized holonomy group. This result may be expected for the reason that the G -structure is a global reduction of the frame bundle with structure group $Spin(1, 4)$ over five-dimensional spacetime to a sub-bundle with structure group G over a base manifold \mathcal{B} .

In section 2 we have shown that the generalized holonomy group for $n = 4$ must be contained in a subgroup $SL(1, \mathbb{H}) \ltimes \mathbb{H}$ of the *generalized* structure group $SL(2, \mathbb{H})$. Recall that $SL(1, \mathbb{H}) \simeq SU(2)$ [13], and furthermore that \mathbb{H} in the semi-direct product can in fact be seen as two doublets $2\mathbb{R}^2$ under $SU(2)$. To see this, recall that a quaternion $Q = q^0 + iq^1 + jq^2 + kq^3 \in \mathbb{H}$, where $q^i \in \mathbb{R}$, can be written as

$$W = \begin{pmatrix} q^0 + iq^3 & q^1 + iq^2 \\ -q^1 + iq^2 & q^0 - iq^3 \end{pmatrix}, \quad (36)$$

where $W \in U(2)$ and $\det(W) = \|Q\|$. The action of $SL(1, \mathbb{H})$ on \mathbb{H} in the semi-direct product is by left multiplication on W , and hence the columns necessarily transform as doublets under $SU(2)$.

This demonstrates that the generalized holonomy for solutions preserving $n = 4$ supersymmetries is in fact $SL(1, \mathbb{H}) \ltimes \mathbb{H} \subset SL(2, \mathbb{H})$, where \mathbb{H} becomes two doublets if the isomorphic group $SL(1, \mathbb{H}) \simeq SU(2)$ is concerned. Thus the solutions of $D = 5$ minimal supergravity support the classification scheme of table 1. In addition, the G -structure is

embedded in the manner $SU(2) \subset SU(2) \ltimes 2\mathbb{R}^2 \simeq SL(1, \mathbb{H}) \ltimes \mathbb{H} \subset SL(2, \mathbb{H})$ for timelike solutions and $\mathbb{R}^3 \subset \mathbb{R}^4 \subset SU(2) \ltimes 2\mathbb{R}^2 \simeq SL(1, \mathbb{H}) \ltimes \mathbb{H} \subset SL(2, \mathbb{H})$ for null solutions. We also notice that $SL(2, \mathbb{H}) \simeq Spin(1, 5)$, which is the structure group of six-dimensional space-time. That is why G -structure only finds a unified picture in six dimensions but not in five dimensions [14].

4.2 More on the generalized holonomy

In the previous section we have seen how the same generalized holonomy group $SU(2) \ltimes 2\mathbb{R}^2$ arises for two different classes of supergravity vacua in five dimensions. It is interesting to see how this works for other theories with 8 supercharges, i.e. $D = 6, \mathcal{N} = (1, 0)$ and $D = 4, \mathcal{N} = 2$ [14, 15]. All three of them are expected to share a common framework from either the G -structure or generalized holonomy points of view. Indeed it has been found that six-dimensional minimal supergravity has only null solutions with G -structure $SU(2) \ltimes \mathbb{R}^4$, which contains the G -structures of the four and five dimensional cases as subgroups [14].

As for the generalized holonomy, for timelike solutions of the four dimensional theory, from the integrability conditions we obtain the complete set of holonomy generators $\{\gamma^{ij}P_-, \gamma^iP_-, \gamma^5P_-\}$, where $i, j = 1, 2, 3$ and $\gamma^5 = \gamma^{0123}, P_- = \frac{1}{2}(1 - \gamma^0)$. In the null case, we have generators $\{\gamma^{i+}P_-, \gamma^+P_-, \gamma^+\gamma^5P_-, \gamma^iP_-, \gamma^5P_-\}$, where $i = 2, 3$ and $\gamma^5 = \gamma^{+-23}, P_- = \frac{1}{2}\gamma^-\gamma^+$. For $D = 6, \mathcal{N} = (1, 0)$, we have generators $\{\gamma^{i+}P_-, \gamma^{ij}P_-\}$, where $i = 2, 3, 4, 5$ and $P_- = \frac{1}{2}\gamma^-\gamma^+$ for only null solutions. Hence for all cases we have the same generalized holonomy group $SU(2) \ltimes 2\mathbb{R}^2 \simeq SL(1, \mathbb{H}) \ltimes \mathbb{H}$ as was expected in section 2. Our results are summarized in table 2 together with the G -structures found in [12, 14]. Depending on the particular solution, the generalized holonomy may be a subgroup of $SU(2) \ltimes 2\mathbb{R}^2$. For example, for solutions with vanishing flux the holonomy group is restricted to the G -structure group which is indeed a subgroup of $SU(2) \ltimes 2\mathbb{R}^2$.

It would be interesting to further test this conjecture on other vacua also with 8 supercharges, such as $D = 4, 5$ supergravity coupled with matter multiplets. Since they can be obtained from $D = 6, \mathcal{N} = (1, 0)$ (without truncation), one may expect their generalized holonomy would be the same as what we found here. Another interesting test could be done with the gauged supergravity vacua [16, 17]. In this case, as we have seen in section 2, the structure group is $SL(4, \mathbb{C})$ and thus it is possible to stabilize 0, 1, 2, 3 or all 4 complex-valued spinors, depending on the solution. This means that a given solution may

Dim	Solution Type	Gen. Structure Group	Generalized Holonomy	G -structure
4	timelike	$SL(2, \mathbb{H})$	$SU(2) \ltimes 2\mathbb{R}^2$	$SU(2)$
4	null	$SL(2, \mathbb{H})$	$SU(2) \ltimes 2\mathbb{R}^2$	\mathbb{R}^2
5	timelike	$SL(2, \mathbb{H})$	$SU(2) \ltimes 2\mathbb{R}^2$	$SU(2)$
5	null	$SL(2, \mathbb{H})$	$SU(2) \ltimes 2\mathbb{R}^2$	\mathbb{R}^3
6	null	$SL(2, \mathbb{H})$	$SU(2) \ltimes 2\mathbb{R}^2$	$SU(2) \ltimes \mathbb{R}^4$

Table 2: Generalized holonomy groups for half-BPS vacua of supergravity with 8 supercharges and the corresponding G -structures.

preserve $n = 0, 2, 4, 6$ or 8 (full) supersymmetry. However, we are reminded that in $D = 11$ supergravity, although all values of preserved supersymmetries from 0 to 32 are all allowed by the M-algebra, not all of them are found [5, 6, 18]. The fact that solutions to gauged $D = 4, 5$ supergravity preserving $n = 6$ are not found [16, 17] may be similar to the conjectured absence of a solution with 31 supersymmetries in conventional $D = 11$ supergravity theory, although an argument for their existence can be made [19].

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